Multiplicative BRST renormalization of the $SU(2)$ Higgs model

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Abstract

We reformulate the proof of the renormalization of a spontaneously broken

gauge theory by multiplicatively renormalizing the vacuum expectation value

of the Higgs field in the $SU(2)$ Higgs model.

After BRST symmetry was discovered [1], it was noted that the renormalization of gauge theories [2] can be simplified by using BRST methods. In [3] Becchi, Rouet and Stora considered the $SU(2)$ Higgs model, introduced sources for the BRST variations of the gauge and ghost fields, and made an analysis of the two-point-functions, the S-matrix, unitarity, and even an cohomologic analysis of anomalies. However, they did not give a systematic loop by loop proof of renormalizability. Also, their work was complicated because the non-nilpotency of the BRST transformations of the antighost introduces ghost equations of motion at various places. At about the same time, Zinn-Justin [4] considered the renormalization of gauge theories loop by loop using BRST Ward identities. For spontaneously broken gauge theories these Ward identities remain valid because they are independent of the value (in particular the sign) of the mass term in the matter section. Using this approach, B.Lee [5] gave a systematic analysis of the multiplicative loop-by-loop renormalization of unbroken gauge theories. This account supercedes his earlier treatments [6] which did not use BRST methods and were much less clear. At the end of his analysis he also briefly considered spontaneously broken gauge theories where he started a proof of the renormalizability of a

general spontaneously broken gauge theory by first shifting the n-loop renormalized scalars s_α^{ren} over an amount δu_α such that tadpoles (one particle irreducible (1PI) 1-point graphs) were removed. This is thus an additive renormalization. The remaining parameters he renormalized multiplicatively. In particular, the scalar fields s_α were renormalized as follows

$$s_\alpha = Z_\alpha^{\frac{1}{2}}(s_\alpha^{\text{ren}} + u_\alpha^{\text{ren}} + \delta u_\alpha) \tag{1}$$

such that s_α^{ren} vanishes at the minimum of the renormalized effective potential. In this note we want to demonstrate that one can treat all renormalizations on equal footing as multiplicative renormalizations. In particular, the vacuum expectation value v gets a Z-factor Z_v which differs from the Z factor of the corresponding scalar field. The advantage of multiplicative instead of additive renormalization is that the BRST symmetry is manifestly preserved under the renormalization program. If one only has additive renormalization, one has to prove this property; such a proof has been given in reference [7].

The model we consider is the $SU(2)$ spontaneously broken gauge theory coupled to the Higgs sector of the standard model [8], with σ the Higgs scalar and χ^a the would-be Goldstone bosons. Surprisingly we find that there is one more divergent structure allowed by the BRST Ward identities than there are Z factors. This problem is resolved because we have found a new identity for the effective action of spontaneously broken gauge theories, which holds in addition to the BRST Ward identities, and which originates from the observation that in the matter sector only the unbroken $\sigma + v$ appears.

The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}(\text{gauge}) + \mathcal{L}(\text{matter}) + \mathcal{L}(\text{fix}) + \mathcal{L}(\text{ghost}) + \mathcal{L}(\text{sources}) \tag{2}$$

where

$$\mathcal{L}(\text{gauge}) = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^a_{bc} A_\mu^b A_\nu^c)^2 \tag{3}$$

$$\mathcal{L}(\text{matter}) = -\frac{1}{2}(D_\mu \sigma)^2 - \frac{1}{2}(D_\mu \chi^a)^2 + \frac{1}{2}\mu^2\{(\sigma+v)^2 + (\chi^a)^2\} - \frac{1}{4}\lambda\{(\sigma+v)^2 + (\chi^a)^2\}^2$$

$$= -\frac{1}{2}(D_\mu \sigma)^2 - \frac{1}{2}(D_\mu \chi^a)^2 - \frac{1}{4}\lambda\{\sigma^2 + (\chi^a)^2\}^2 - \lambda v\sigma[\sigma^2 + (\chi^a)^2]$$

$$-\lambda v^2 \sigma^2 - \beta v \sigma - \frac{1}{2}\beta(\sigma^2 + (\chi^a)^2) \tag{4}$$

$$\mathcal{L}(\text{fix}) = -\frac{1}{2\alpha}(\partial^\mu A_\mu^a - \xi(\frac{1}{2}gv)\chi^a)^2 \tag{5}$$

$$\mathcal{L}(\text{ghost}) = b_a\{\partial^\mu D_\mu c^a - \xi\frac{1}{2}gv(\frac{1}{2}g(\sigma + v)c^a + \frac{1}{2}gf^a_{bc}\chi^b c^c)\} \tag{6}$$

$$\mathcal{L}(\text{sources}) = K_a^\mu D_\mu c^a - K(\frac{1}{2}g\chi_a c^a) + K_a(\frac{1}{2}g(\sigma + v)c^a + \frac{1}{2}gf^a_{bc}\chi^b c^c) + L_a(\frac{1}{2}gf^a_{bc}c^b c^c) \tag{7}$$

The parameter β is given by

$$\beta = -\mu^2 + \lambda v^2$$

and $-\frac{1}{2}\mu^2((\sigma + v)^2 + (\chi^a)^2)$ is the mass-term in the matter section before spontaneous symmetry breaking. Since μ^2 and λv^2 will in general renormalize differently, one cannot expect that β renormalizes multiplicatively. It is very convenient to require that the value of the renormalized β be zero

$$\beta_{\text{ren}} = -\mu_{\text{ren}}^2 + \lambda_{\text{ren}}v_{\text{ren}}^2 = 0 \tag{8}$$

since this eliminates terms linear in σ from the quantum action. Of course, $\beta_{\text{ren}} = 0$ also excludes multiplicative renormalizability of β. In the preceding article [9] we found it useful to consider β instead of μ^2 as an independent variable, and renormalized β additively. Taking now μ^2 as independent variable saves multiplicative renormalization.

The external sources K, K_a and K_a^μ multiply the BRST variations of σ, χ^a and A_μ^a, and the theory with (and hence without) them will be shown to be renormalizable. The covariant derivatives are given by

$$D_\mu \sigma = \partial_\mu \sigma + \frac{1}{2}gA_\mu^a \chi_a \tag{9}$$

$$D_\mu \chi^a = \partial_\mu \chi^a - \frac{1}{2}gA_\mu^a(\sigma + v) + \frac{1}{2}gf^a_{bc}A_\mu^b \chi^c. \tag{10}$$

Clearly, $\mathcal{L}(\text{matter})$ depends only on $\sigma + v$, but $\mathcal{L}(\text{fix})$ and $\mathcal{L}(\text{ghost})$ violate this property for $\xi \neq 0$. Hence we may expect that σ and v will renormalize differently if $\xi \neq 0$. We shall assume that the renormalized ξ and α have 't Hooft's [8] values $\xi_{\text{ren}} = \alpha_{\text{ren}} = 1$ in order that the propagators be diagonal and simple.

The two Ward identities used by B.Lee [5] for the effective action Γ read before renormalization

$$\partial\tilde{\Gamma}/\partial\Phi^I \frac{\partial}{\partial K^I}\tilde{\Gamma} = 0 \tag{11}$$

$$(\partial^\mu \frac{\partial}{\partial K_a^\mu} - \xi\frac{1}{2}gv\frac{\partial}{\partial K_a} - \frac{\partial}{\partial b_a})\tilde{\Gamma} = 0 \tag{12}$$

where $\Phi^I = \{\sigma, \chi^a, A_\mu^a, c^a\}$, $K^I = \{K, K_a, K_a^\mu, L_a\}$ and $\tilde{\Gamma} = \Gamma - \int \mathcal{L}(\text{fix})d^4x$. In addition we shall use below two further identities related to ghost number conservation and to the symmetry of $\mathcal{L}(\text{matter})$ under $\sigma \to \sigma + \Delta v, v \to v - \Delta v$.

The Ward identities in (11) and (12) remain valid after renormalization if all rescalings are such that they amount to an overall factor. Choosing $A_\mu^a = (Z_3)^{\frac{1}{2}}A_{\mu,\text{ren}}^a$ and $c^a = (Z_{\text{gh}})^{\frac{1}{2}}c_{\text{ren}}^a$, and furthermore $\sigma = (Z_\sigma)^{\frac{1}{2}}\sigma_{\text{ren}}$ and $\chi^a = (Z_\chi)^{\frac{1}{2}}\chi_{\text{ren}}^a$ (Z_χ is independent of the $SU(2)$ index a since $\mathcal{L}(\text{fix})$ is $SU(2)$ invariant), we assume, to be proven by induction in order of loops, the following properties:

1. $\tilde{\Gamma}$ is made finite by multiplicative rescalings of all objects. In particular, K_a^μ and b_a scale like c^a, while L_a scales like A_μ^a. Furthermore the scales of K and K_a are such that σK and $\chi^a K_a$ have the same Z factor as $A_\mu^a K_a^\mu$.

2. α and ξ must scale such that $\mathcal{L}(\text{fix})$ is finite by itself since we now deal with $\tilde{\Gamma}$ which is Γ minus $\int \mathcal{L}(\text{fix})d^4x$.

We also renormalize $g = Z_g g_{\text{ren}}, v = (Z_v)^{\frac{1}{2}}v_{\text{ren}}, \lambda = Z_\lambda\lambda_{\text{ren}}$ and $\mu^2 = Z_{\mu^2}\mu_{\text{ren}}^2$. Hence

$$K = (Z_3 Z_{\text{gh}}/Z_\sigma)^{\frac{1}{2}}K_{\text{ren}}$$

$$K^a = (Z_3 Z_{\text{gh}}/Z_\chi)^{\frac{1}{2}}K_{\text{ren}}^a$$

$$\xi = Z_3^{\frac{1}{2}}Z_g^{-1}Z_v^{-\frac{1}{2}}Z_\chi^{-\frac{1}{2}}\xi_{\text{ren}} \tag{13}$$

The equality of the Z-factors of b and c is not a matter of choice because in $\mathcal{L}(\text{source})$ there are terms with c but without b.

It is instructive to do a quick one-loop analysis of the σ^4 and σ^3 1PI Green's functions to convince oneself that Z_v is not equal to Z_σ. In figure 1 we have given the coefficients of the

divergences of the relevant divergent graphs. Clearly four times the sum of the first three coefficients does not equal the sum of the last two coefficients which shows that $Z_v \neq Z_\sigma$ in the gauge sector. Since in the matter sector $Z_v = Z_\sigma$ (the χ mass from $\mathcal{L}(\text{fix})$ does not change this result since massless tadpoles cancel each other without having to assume that $\int \frac{d^4 k}{k^2} = 0$, see [9]), we see that in the model given by (2) one has $Z_v \neq Z_\sigma$. We expect $Z_v = Z_\sigma$ if $\xi = 0$. Computations with $\xi = 0$ are somewhat complicated because propagators are off-diagonal. The Z-factors for the σ and χ^a fields are the same because the terms in the action of dimension 4 have a $SO(4)$ symmetry, even after gauge fixing. (In the notation of reference [8], the $SU(2)$ gauge transformation acts from the left on $D_\mu(\sigma + i\chi^a\tau_a)$, while the other $SU(2)$ group is rigid, acting from the right leaving A_μ^a, c^a and b_a invariant).

Assuming $(n-1)$-loop finiteness of $\tilde{\Gamma}$ (and hence of Γ), the n-loop 1PI divergences satisfy the equations

$$Q_{\text{ren}} \tilde{\Gamma}_{\text{ren}}^{(n),\text{div}} = 0 \tag{14}$$

where $Q_{\text{ren}} = \partial \tilde{\Gamma}_{\text{ren}}^{(0)}/\partial \Phi_{\text{ren}}^I \frac{\partial}{\partial K_I^{\text{ren}}} - \partial \tilde{\Gamma}_{\text{ren}}^{(0)}/\partial K_I^{\text{ren}} \frac{\partial}{\partial \Phi_{\text{ren}}^I}$ and

$$(\partial^\mu \frac{\partial}{\partial K_{a,\text{ren}}^\mu} - \xi_{\text{ren}}(\frac{1}{2}gv)_{\text{ren}} \frac{\partial}{\partial K_a^{\text{ren}}} - \frac{\partial}{\partial b_a^{\text{ren}}}) \tilde{\Gamma}_{\text{ren}}^{(n),\text{div}} = 0 \tag{15}$$

where $\tilde{\Gamma}_{\text{ren}}^{(0)}$ equals the quantum action minus $\int \mathcal{L}(\text{fix})d^4x$, all in terms of objects multiplicatively renormalized such that all 1PI graphs with $(n-1)$ loops are finite. We shall drop the subscripts "ren", understanding that from now on all objects are $(n-1)$ loop renormalized.

The n-loop divergences are local, and (15) states that b_a can only appear in the divergences in the combination $K_a^\mu - \partial^\mu b_a$ or $K_a - \xi\frac{1}{2}gvb_a$. This excludes divergences proportional to $b_a\partial^\mu A_\mu^a$, $vb_a\chi^a$ or $b_a\sigma\chi^a$. The general form of the n-loop divergences is given by

$$\tilde{\Gamma}_{\text{ren}}^{(n),\text{div}} = \sum_{i=1}^4 a_i(\epsilon)G^i + Q_{\text{ren}} \sum_{j=1}^5 b_j(\epsilon)X^j \tag{16}$$

where the first term contains all possible gauge-invariant local expressions, see (3) and (4),

$$\sum a_i G^i = a_1 S(\text{gauge}) + a_2 S(\text{kin. matter}) + a_3 S(\text{mass matter}) + a_4 S(\text{pot.}) \tag{17}$$

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while the second term is given by

$$\sum b_j X^j = b_1(K_a^\mu - \partial^\mu b_a)A_\mu^a + b_2(K_a - \frac{1}{2}gv\xi b_a)\chi^a + b_3 K\sigma + b_4 L_a c^a + b_5 Kv. \quad (18)$$

Because of the $SO(4)$ symmetry, $b_2 = b_3$, but we shall keep writing b_2 and b_3 separately in order to facilitate the identification of divergences. It is easy to see that (16) is a solution of (14) and (15) since Q, the BRST charge, acting on a gauge invariant term is zero and $Q^2 = 0$ (see [5] and [10]). In [11] a general (model independent) but rather complicated (and incomplete) proof is given that the general solution of (14) is a sum of gauge invariant terms and Q-exact terms as in (16). It is possible to prove this for a given model in a simple and direct way as follows:

1. write down all local expressions with dimension four and ghost-number zero which can be a priori divergent according to power counting

2. use the fact that their sum must be annihilated by Q_{ren}.

For the model in (2), the result is (16).

We observe that there are eight divergent structures but only seven Z-factors (for A_μ^a, σ and $\chi^a, c^a, g, v, \lambda, \mu^2$). In pure unbroken Yang-Mills theory there is no such mismatch, but in the matter coupled case with unbroken symmetry the same mismatch occurs. As we shall see, multiplicative renormalizability is still possible because the eight divergences a_i and b_j (where $b_2 = b_3$) only occur in seven combinations.

To prove multiplicative renormalizability, each of the local divergences should be written as a counting operator $x\frac{\partial}{\partial x}$ acting on $\tilde{\Gamma}^{(0),\text{ren}}$ where x denotes all fields, sources and parameters in the theory. For most terms, the analysis has already been given by B.Lee [5]. In particular

$$S(\text{gauge}) = \frac{1}{g^2}S(gA_\mu^a) \quad (19)$$
$$= (-\frac{1}{2}g\frac{\partial}{\partial g} + \frac{1}{2}A_\mu^a\frac{\partial}{\partial A_\mu^a} + \frac{1}{2}L_a\frac{\partial}{\partial L_a}$$

$$+\frac{1}{2}K\frac{\partial}{\partial K}+\frac{1}{2}K_a\frac{\partial}{\partial K_a}+\xi\frac{\partial}{\partial\xi})\tilde{\Gamma}_{\text{ren}}^{(0)} \tag{20}$$

$$S(\text{kin. matter}) = (\frac{1}{2}\sigma\frac{\partial}{\partial\sigma}+\frac{1}{2}v\frac{\partial}{\partial v}+\frac{1}{2}\chi_a\frac{\partial}{\partial\chi_a}-\mu^2\frac{\partial}{\partial\mu^2}$$

$$-2\lambda\frac{\partial}{\partial\lambda}-\frac{1}{2}K\frac{\partial}{\partial K}-\frac{1}{2}K_a\frac{\partial}{\partial K_a}-\xi\frac{\partial}{\partial\xi})\tilde{\Gamma}_{\text{ren}}^{(0)}. \tag{21}$$

The terms from QX lead to the counting operators

$$b_1(A_\mu^a\frac{\partial}{\partial A_\mu^a}-(K_a^\mu-\partial^\mu b_a)\frac{\partial}{\partial K_a^\mu})+b_2(\chi_a\frac{\partial}{\partial\chi_a}-(K_a-\frac{1}{2}gv\xi b_a)\frac{\partial}{\partial K_a})$$

$$+b_3(\sigma\frac{\partial}{\partial\sigma}-K\frac{\partial}{\partial K})+b_4(c_a\frac{\partial}{\partial c_a}-L_a\frac{\partial}{\partial L_a})+b_5 v\frac{\partial}{\partial\sigma}. \tag{22}$$

Most terms in QX are already of the form $x\frac{\partial}{\partial x}\tilde{\Gamma}_{\text{ren}}^{(0)}$. We now analyze the terms which are not yet cast into this form

$$(b_1\partial^\mu b_a\frac{\partial}{\partial K_a^\mu}+b_2\xi\frac{1}{2}gvb_a\frac{\partial}{\partial K_a}+b_5 v\frac{\partial}{\partial\sigma})\tilde{\Gamma}_{\text{ren}}^{(0)}. \tag{23}$$

The first term equals $-S(\text{ghost})$ at $\xi=0$, and can be written as $-\frac{1}{2}b_a\frac{\partial}{\partial b_a}-\frac{1}{2}c_a\frac{\partial}{\partial c_a}+\frac{1}{2}K\frac{\partial}{\partial K}+\frac{1}{2}K_a\frac{\partial}{\partial K_a}+\frac{1}{2}K_a^\mu\frac{\partial}{\partial K_a^\mu}+L_a\frac{\partial}{\partial L_a}+\xi\frac{\partial}{\partial\xi}$ acting on $\tilde{\Gamma}_{\text{ren}}^{(0)}$. The second term is minus the ξ term in $S(\text{ghost})$, hence it equals $-\xi\frac{\partial}{\partial\xi}$ acting on $\tilde{\Gamma}_{\text{ren}}^{(0)}$. The last term we deal with later.

Analyzing these results, we see that the combination $c_a\frac{\partial}{\partial c_a}+b_a\frac{\partial}{\partial b_a}$ appears everywhere except in the term with b_4. However, ghost number conservation leads to the Ward identity

$$(b_a\frac{\partial}{\partial b_a}-c_a\frac{\partial}{\partial c_a}+K\frac{\partial}{\partial K}+K_a\frac{\partial}{\partial K_a}+K_a^\mu\frac{\partial}{\partial K_a^\mu}+2L_a\frac{\partial}{\partial L_a})\tilde{\Gamma}_{\text{ren}}^{(0)}=0 \tag{24}$$

and using this identity to convert half of the b_4 terms, we find also in the b_4 term the desired combination $c_a\frac{\partial}{\partial c_a}+b_a\frac{\partial}{\partial b_a}$. At this point the divergences can be written as

$$\tilde{\Gamma}_{\text{ren}}^{(n),\text{div}} = [\ (\frac{1}{2}a_1+b_1)(A_\mu^a\frac{\partial}{\partial A_\mu^a}+L_a\frac{\partial}{\partial L_a})$$

$$+(-\frac{1}{2}b_1+\frac{1}{2}b_4)(c_a\frac{\partial}{\partial c_a}+b_a\frac{\partial}{\partial b_a}+K_a^\mu\frac{\partial}{\partial K_a^\mu})$$

$$+(\frac{1}{2}a_2+b_3)\sigma\frac{\partial}{\partial\sigma}+(\frac{1}{2}a_2+b_2)\chi_a\frac{\partial}{\partial\chi_a}$$

$$+(\frac{1}{2}a_1-\frac{1}{2}a_2+\frac{1}{2}b_1-b_3+\frac{1}{2}b_4)K\frac{\partial}{\partial K}$$

$$+(\frac{1}{2}a_1-\frac{1}{2}a_2+\frac{1}{2}b_1-b_2+\frac{1}{2}b_4)K_a\frac{\partial}{\partial K_a}$$

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$$-\frac{1}{2}a_1 g\frac{\partial}{\partial g} + (a_1 - a_2 + b_1 - b_2)\xi\frac{\partial}{\partial \xi}$$
$$+ (-a_2 + a_3)\mu^2\frac{\partial}{\partial \mu^2} + (-2a_2 + a_4)\lambda\frac{\partial}{\partial \lambda}$$
$$+ \frac{1}{2}a_2 v\frac{\partial}{\partial v} + b_5 v\frac{\partial}{\partial \sigma}]\tilde{\Gamma}^{(0)}_{\text{ren}}. \tag{25}$$

Since $b_2 = b_3$, we see that indeed $Z_\sigma = Z_\chi$ and the Z-factors for K and K_a are equal. We also see that A^a_μ and L_a scale the same way as do c^a, b_a and K^μ_a. Furthermore, the factors in front of $K\frac{\partial}{\partial K}$ (or $K_a\frac{\partial}{\partial K_a}$) depend linearly on those corresponding to A^a_μ, c^a and σ (or χ^a), namely in agreement with (13).

As usual, $\alpha = Z_3\alpha_{\text{ren}}$ fulfills step 2 of the induction as far as the ξ-independent terms are concerned. We are left with the only nontrivial part of the proof of renormalizability, the proof that the rescaling of ξ in (13) is consistent with the rescaling of v which has been left unspecified so far. The key to this compatibility lies in the last term in (25), the term with $b_5 v\frac{\partial}{\partial \sigma}\tilde{\Gamma}^{(0)}_{\text{ren}}$. To write it down, too, as a counting operator, we recall that the matter action is annihilated by $v\frac{\partial}{\partial \sigma} - v\frac{\partial}{\partial v}$ (since it only depends on $\sigma + v$). There is only one place in $\tilde{\Gamma}^{(0)}_{\text{ren}}$ where v appears separately, namely in the term with ξ in $\mathcal{L}(\text{ghost})$. (Recall that in $\tilde{\Gamma}$ there is no $\mathcal{L}(\text{fix})$). Clearly then, the following identity holds

$$(\xi\frac{\partial}{\partial \xi} - v\frac{\partial}{\partial v} + v\frac{\partial}{\partial \sigma})\tilde{\Gamma}^{(0)}_{\text{ren}} = 0. \tag{26}$$

Using this identity to eliminate $v\frac{\partial}{\partial \sigma}$ from (25), we find that v rescales with $\frac{1}{2}a_2 + b_5$ and ξ with $(a_1 - a_2 + b_1 - b_2 - b_5)$. These renormalizations of ξ and v are then also in agreement with the rescaling of ξ in (13). Hence the multiplicative renormalizability of the spontaneously broken $SU(2)$ Higgs model is proven.

ACKNOWLEDGMENTS

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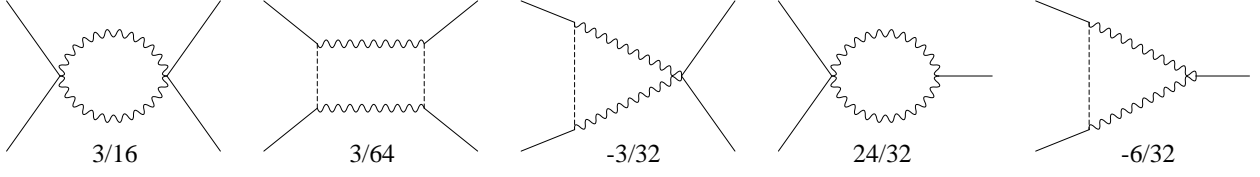
Wiggly lines denote gauge fields, dotted lines denote would-be Goldstone bosons

FIG. 1.   The σ^4 and σ^3 one-loop vertex corrections